TONBRIDGE SCHOOL

Scholarship Examination 2014

MATHEMATICS I

Monday 28th April 2014
11.30 am

Time allowed: 1 hour 30 minutes

Answer as many questions as you can.
Questions 1 to 5 are worth 8 marks each;
Questions 6 to 9 are worth 15 marks each.

All answers must be supported by adequate explanation.
Calculators may be used in any question.

1. Find the length and width of a rectangle that has the following properties:
   - The length is 5 cm greater than the width;
   - The perimeter of the rectangle is 24 cm.

   [8 marks]

2. (a) Find the area of a circle with circumference 100 cm.

   (b) Find the circumference of a circle with area 100 cm².

   [8]

3. For a tall building of height H metres, the difference in the time of sunrise (T minutes) between the top and bottom of the building is given by the formula $T = 10.2\sqrt{\dfrac{H}{R}}$, where R is the radius of the Earth in kilometres.

   (a) Find T for the Shard building in London where $H = 309.6$ and $R = 6400$.

   (b) Find H for the CN Tower in Toronto where $T = 3.0$ and $R = 6400$.

   (c) For the Empire State Building in New York, $H = 381$ and $T = 2.55$. What value do these figures give for the radius of the Earth?

   [8]

4. The first term of a sequence is 2 and the second term is 5. Subsequent terms are given by the rule:
$$\text{NEXT TERM} = \frac{1 + \text{ CURRENT TERM}}{\text{PREVIOUS TERM}}$$

   (a) Show that the third term is 3 and find the 4th, 5th, 6th, 7th terms.

   (b) What do you notice about your answers in part (a)?

   (c) Use part (b) to find:
       (i) The 59th term of the sequence;
       (ii) The sum of the first 147 terms of the sequence.

   [8]

5. A square and a regular pentagon share a common edge as shown in the figure below.
    Points A, B, C are three vertices of another regular polygon. How many sides does it have?
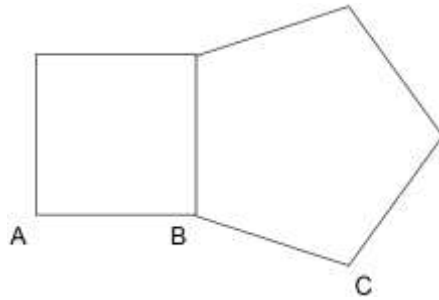

    [8]

6. A certain species of bamboo plant increases its height by 15% every year.

    (a)   A bamboo plant is 5 m tall.
          (i) How tall will it be in a year's time?
          (ii) How tall will it be in two years' time?
          (iii) After how many years will the bamboo first exceed 10 m in height?

    (b)   A second bamboo plant is 12.2 m tall. How tall was it when it was planted 1 year ago?
          (You should do this part by a calculation not trial and improvement.)

    (c)   A third bamboo plant has increased in height by 1.29 m since it was planted 2 years ago.
          How tall is it now?

    [15]

7. If the right-angled triangle in the diagram below is rotated through $360°$ about its hypotenuse of
    fixed length 6 cm, the double-cone formed has volume y cm³ given by the formula
    $$y = \frac{\pi x^2 (36-x^2)}{18},$$ where x is the side-length shown and $\pi = 3.14159...$.
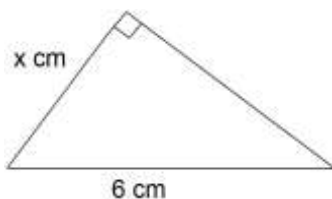

    (a)   When $x = 2$, show that $y = 22.3$ (correct to 3 significant figures).

    (b)   Find the values of y for $x = 0, 1, 2, 3, 4, 5, 6$.

    (c)   Choosing sensible scales, use your values to plot a graph of y against x.

    (d)   What is the maximum volume possible for the double-cone?

    (e)   What is special about the shape of the triangle which gives rise to the maximum volume
          in part (d)?

    [15]

8. The figure shows a kite with a vertical axis of symmetry and two angles and one side-length as shown.
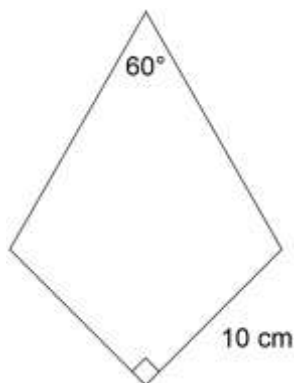
(a) Find the perimeter of the kite.

(b) Find the area of the kite.

[15]

9. Study carefully the pattern of numbers in the table below.
Column A shows a calculation with fractions that results in the first two numbers in Column B.
The third number in Column B comes from the calculation shown in Column C.

	A	B	C
Row 1	$1\frac{1}{3} = \frac{4}{3}$	3, 4, 5	$3^2 + 4^2 = 5^2$
Row 2	$2\frac{2}{5} = \frac{12}{5}$	5, 12, 13	$5^2 + 12^2 = 13^2$
Row 3	$3\frac{3}{7} = \frac{24}{7}$	7, 24, 25	$7^2 + 24^2 = 25^2$
Row 4			
Row 5			
Row n			

(a) What are the entries in Columns A, B, C for Rows 4 and 5?

(b) If the first number in Column B is 23, what is the entry in Column A?

(c) If the second number in Column B is 840, what is the entry in Column A?

(d) If the third number in Column B is 685, what is the entry in Column A?

(e) What is the entry in Column A for Row n?

[15]

END OF PAPER